$$\Lambda : (v_1, \dots, v_n) \longrightarrow \Lambda(v_1, \dots, v_n)$$

$$(1,2,3,4) \longrightarrow \Lambda(1,2,3,4)$$

$$\Lambda(1,2,3,4) \neq \Lambda(1,3,2,4)$$

$$(12)\Lambda : (v_1, \dots, v_n) \longrightarrow \Lambda(v_2, v_1, \dots, v_n)$$

$$(ij)\Lambda : (v_1, \dots, v_i \dots v_j \dots, v_n) \longrightarrow \Lambda(v_1, \dots, v_j, \dots, v_i, \dots v_n)$$

$\sigma \in \mathfrak{S}_n$ = permutation de $\{1, \dots, n\}$

$\sigma . \Lambda : (v_1, \dots, v_n) \longrightarrow \Lambda(v_{\sigma(1)}, v_{\sigma(2)}, \dots, v_{\sigma(n)})$

- Λ est symetrique si $\forall i, j \quad (ij)\Lambda = \Lambda$
  altenee si $\forall i, j \quad (ij)\Lambda = -\Lambda$
- Λ est symetrique ssi $\forall \sigma \in \mathfrak{S}_n \quad \sigma . \Lambda = \Lambda$

– si Λ est alternée $\forall \sigma \in \mathfrak{S}_n$

$$\sigma . \Lambda = \mathrm{sign}(\sigma) \Lambda$$

$$(1,2)\big((1\,2)\,\Lambda\big) = \Lambda(v_1, v_2, v_3 \dots v_n)$$

$$\det(v_1, \dots, v_d) = \sum_{\sigma \in \mathfrak{S}_d} \operatorname{sign}(\sigma)\, x_{1\sigma(1)} \times \dots \times x_{d\sigma(d)}$$

$d = 2 \qquad v_1 = (x_{11}, x_{12}) \qquad v_2 = (x_{21}, x_{22})$

$\mathfrak{S}_2 = \{ Id_2, (12) \} \qquad \operatorname{sign}(I_2) = 1 \quad \operatorname{sign}(12) = -1$

$$\det((x_{11}, x_{12}), (x_{21}, x_{22})) = \underbrace{1\, x_{11} x_{22}}_{\sigma = I_2} + \underbrace{(-1) x_{12} x_{21}}_{\sigma = (12)}$$

$$\det_2(v_1, v_2) = x_{11}x_{22} - x_{12}x_{21}$$

$d = 3$ $\quad \mathfrak{S}_3 = \{ I_3, (12), (13), (23), (123), (132) \}$

$$\det_3(v_1, v_2, v_3) = 1 x_{11}x_{22}x_{33} - x_{12}x_{21}x_{33} - x_{13}x_{22}x_{31} - x_{11}x_{23}x_{32} + x_{12}x_{23}x_{31} + x_{13}x_{21}x_{32}$$